Here $\mathcal{B}_i$ denotes the set of ensembles $B \in \mathcal{B}_V$ with $f(v, B) = i$ for some $v \in V$.

Restricting Over-Supply As we have already pointed out in Section 3, over-supply is sometimes essential if we want to minimize the overall number of required frequency blocks. However, over-supply also incurs a cost since it will usually increase the number of ensembles per area and hence require a larger number of transmitters to realize the ensemble assignment. In order to restrict the amount of over-supply we might put a limit on the number of ensembles for a given area. Another possible approach would be to charge for the bandwidth allocated to a service. E.g., we might have a cost γ_{vs} associated with each area $v \in V$ and service $s \in S$ which can be thought of as the prize the service provider has to pay (or the negative net income he earns) for supplying the service in the given area. The total cost of an ensemble assignment $\mathcal{B}$ could then be defined as

$$g(\mathcal{B}) = \sum_{(v,B)\in\mathcal{B}} \sum_{s\in B} \gamma_{vs}. \tag{13}$$

Reducing the total cost of an ensemble assignment will then also reduce over-supply. An obvious way to bring the cost function into the main objective function is to have another prize α on each frequency block required by the block assignment, which allows us to adjust the weights between the two objectives. Adding both partial cost functions together we obtain a new objective function

$$h(\mathcal{B}) = \alpha|f(\mathcal{B})| + g(\mathcal{B}), \tag{14}$$

which might be used as a heuristic valuation function in some local improvement heuristic.

Another type of constraint, which frequently arises in practice, is that for certain economical or political reasons we might wish to specify which services must *not* be supplied in a given area. We call this the ensemble planning problem *with forbidden services.* (Actually this variation of the problem can also be interpreted as a special case of the "bandwidth prizing" discussed above, since we can always choose the prizes in a manner which makes it prohibitively expensive to supply a service in a certain area.)

To formulate this problem, we extend the problem description with sets $T_v : v \in V$ of forbidden services. An admissible ensemble assignment will then be required to also satisfy the constraint

$$T_v \cap \bigcup_{B\in\mathcal{B}_v} B = \emptyset \;\forall\, v \in V. \tag{15}$$

Unfortunately, the ensemble planning problem with forbidden services appears to be much harder than the ordinary ensemble planning problem. To see why, consider an arbitrary graph $H = (V, E)$. From this graph we can construct an instance of the ensemble planning problem with forbidden services as follows. We let G be the complete graph on V. As our service set we choose $S = V$, where $\mu_v = 1 \;\forall\; v \in V$ and $M = |V|$, so that all services fit into a single ensemble. Finally we let $R_v = \{v\}$ and $T_v = \{w \in V :$