$vw \in E\} \ \forall \ v \in V$. Since G is complete, we have that $\chi(G^{\mathcal{B}}) = |\mathcal{B}_V|$ for each ensemble assignment $\mathcal{B}$. Moreover, it is easy to see that the minimum number of ensembles in an admissible ensemble assignment, which also satisfies the forbidden services constraints, is just $\chi(H)$. This shows that the ensemble planning problem with forbidden services, even when restricted to complete graphs, is just as hard as the (general) graph coloring problem. In particular, approximating this special case of the problem is NP-hard, whereas the ordinary ensemble packing problem on complete graphs reduces to bin packing, which can be approximated in polynomial time quite easily.

9 Conclusion

This research is still in progress, and the ensemble planning algorithms described in this paper are just a beginning. In particular, Figures 3 and 4 indicate that at least for random instances on UD graphs, there appears to be a "critical range" of the number of services, which depends on the number of areas and the density of the area graph, for which both the SFF and GFF solutions still leave much room for further improvements. As the space of candidate solutions for this problem is so large (its size grows doubly exponential w.r.t. the number of services), exact algorithms based on branch-and-bound methods are out of question for the problem sizes arising in practice, and so we will probably have to resort to some kind of local improvement heuristic if we want to find better solutions in a reasonable amount of time. We are currently investigating tabu search techniques with which we want to further improve the solution quality and address some of the generalizations discussed in Section 8. Just as with the simple algorithms presented in this paper, the basic idea is to decompose the problem into a packing and a coloring stage, but we also add a "feedback loop" to successively improve the current ensemble assignment w.r.t. the number of required frequency blocks and possibly some other, secondary objectives.

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