In the following, for given G, R and M, by $\chi_M^R(G)$ we denote the minimum number of frequency blocks in any admissible solution. It is not difficult to see that the admissible block assignments f for a given ensemble assignment $\mathcal{B}$ actually are in one-to-one correspondence to the valid colorings of an associated *ensemble graph* $G^{\mathcal{B}}$ which has $\mathcal{B}$ as its set of vertices and whose edges are precisely those pairs $(v, B)(w, C)$ for which $B \neq C \wedge (v = w \vee vw \in E)$; cf. Equation (3). Consequently we have that

$$\chi_M^R(G) = \min\{\chi(G^{\mathcal{B}}) : \mathcal{B} \text{ admissible}\}. \tag{4}$$

Hence the ensemble planning problem is nothing but a graph coloring problem on top of a kind of packing problem. We further explore this in the following sections.

5 Solution Techniques

It is not difficult to see that the ensemble planning problem is NP-complete; in fact it contains both the graph coloring and the bin packing problem as special cases. (For the graph coloring problem, take $S = V$, $R_v = \{v\}\ \forall\ v \in V$ and $\mu \equiv 1 = M$; for the bin packing problem, let G be a one-vertex graph.) So we know that Problem 1 is not only NP-complete, but also difficult to approximate, and hence we will be interested in heuristic solutions.

How can we approach this problem? We have already mentioned that the problem reduces to an ordinary graph coloring problem once we have obtained a suitable ensemble assignment. Thus a straightforward approach is to solve the problem in two optimization stages:

- Find an admissible ensemble assignment $\mathcal{B}$.
- Color $G^{\mathcal{B}}$ using some heuristic graph coloring procedure.

For the second stage, a plethora of different graph coloring algorithms is already available. We can simply plug any of these methods into our ensemble planning procedure. In fact, it turns out that fairly simple "sequential" coloring methods like Brélaz' DSATUR [1] or Matula/Beck's smallest-last algorithm [12] usually perform very well on the kind of "geometric" graphs arising as models of broadcast networks. So we can concentrate on the first step, which obviously is a kind of simultaneous bin packing problem. The difficult part here is to devise a packing strategy which reduces the global number of required *colors* instead of merely optimizing the packings for individual areas. Of course no single strategy will work with all area graphs equally well, so let us take a look at the two extreme cases:

- *Independent (edgeless) graphs.* In this case the problem decomposes into $|V|$ independent bin packing problems, one for each vertex of G, and we have that $\chi_M^R(G) = \max\{p_M(R_v) : v \in V\}$.
- *Complete graphs.* In this case, all distinct ensembles will have to be assigned different frequency blocks, hence $\chi_M^R(G) = p_M(R_V)$.