- the *requirements* R_v, $v \in V$, which denote, for each area $v \in V$, the set of services to be supplied in that area;
- the maximum *ensemble size* $M > 0$.

A solution to the ensemble planning problem consists of two items, an *ensemble assignment* and a corresponding *block assignment*. An *ensemble assignment* is a relation $\mathcal{B} \subseteq V \times 2^S$, which assigns to each $v \in V$ a set $\mathcal{B}_v = \{B : (v, B) \in \mathcal{B}\}$ of *ensembles* (service sets) which are to be transmitted in the corresponding area. For an ensemble assignment $\mathcal{B}$ to be *admissible*, it must satisfy the supply requirements, and the individual ensembles must not exceed the maximum ensemble size:

$$R_v \subseteq \bigcup_{B \in \mathcal{B}_v} B \; \forall \, v \in V, \tag{1}$$

$$\mu_B \leq M \; \forall \, B \in \mathcal{B}_v, v \in V, \tag{2}$$

where the total bandwidth μ_B of an ensemble $B \subseteq S$ is defined by $\mu_B = \sum_{s \in B} \mu_s$.

The second part of the solution is the *block assignment* f, which maps each $(v, B) \in \mathcal{B}$ to a corresponding frequency block or "color" $f(v, B)$. To be *admissible*, the block assignment must not introduce any interferences, i.e., different ensembles in the same or interfering areas must always be assigned different frequency blocks:

$$f(v, B) \neq f(w, C) \; \forall \, (v, B), (w, C) \in \mathcal{B} : B \neq C \wedge (v = w \vee vw \in E). \tag{3}$$

Finally, the *target function* to be minimized is the number of distinct frequency blocks, i.e., $|f(\mathcal{B})| = |\{f(v, B) : (v, B) \in \mathcal{B}\}|$. We can now formulate the ensemble planning problem in the usual decision format as follows:

Problem 1 (Ensemble Planning Problem) Given $G = (V, E)$, $R : V \mapsto 2^S$, $M \in \mathbb{N}$ and $K \in \mathbb{N}$, decide whether there is an admissible ensemble assignment $\mathcal{B}$ and corresponding admissible block assignment f s.t. $|f(\mathcal{B})| \leq K$.

In the following, for given G, R and M, by $\chi_M^R(G)$ we denote the minimum number of frequency blocks in any admissible solution. It is not difficult to see that the admissible block assignments f for a given ensemble assignment $\mathcal{B}$ actually are in one-to-one correspondence to the valid colorings of an associated *ensemble graph* $G^{\mathcal{B}}$ which has $\mathcal{B}$ as its set of vertices and whose edges are precisely those pairs $(v, B)(w, C)$ for which $B \neq C \wedge (v = w \vee vw \in E)$; cf. Equation (3). Consequently we have that

$$\chi_M^R(G) = \min\{\chi(G^{\mathcal{B}}) : \mathcal{B} \text{ admissible}\}. \tag{4}$$

Hence the ensemble planning problem is nothing but a graph coloring problem on top of a kind of packing problem. We further explore this in the following.

4 Solution Techniques

It is not difficult to see that the ensemble planning problem is NP-complete; in fact it contains both the graph coloring and the bin packing problem as special cases. (For the graph coloring problem, take $S = V$, $R_v = \{v\} \; \forall \, v \in V$ and $\mu \equiv 1 = M$; for the bin packing problem, let