

coloring and the clique problem are NP-complete. Moreover, both problems are also difficult to *approximate*: the problem to determine a coloring using at most $|V|^{1/7-\varepsilon}\chi(G)$ colors, and the problem to find a clique of size at least $|V|^{\varepsilon-1/4}\omega(G)$, are NP-hard for each $\varepsilon > 0$ [3]. This implies, in particular, that it is not possible to approximate the optimization versions of these problems within constant performance ratio in polynomial time, unless $P = NP$.

Another type of NP-complete problem we deal with is the *bin packing problem*. Given a set X with positive element sizes μ_x , $x \in X$, and $M > 0$, an M -packing of X is a set \mathcal{B} of mutually disjoint subsets of X s.t. $X = \bigcup_{B \in \mathcal{B}} B$ and the total size $\mu_B = \sum_{x \in B} \mu_x$ of each “bin” $B \in \mathcal{B}$ is at most M . The bin packing problem is, given X , μ , M and a nonnegative integer k , to decide whether X has an M -packing \mathcal{B} of size at most k . We also let $p_M(X)$ denote the minimum size of an M -packing of X , called the M -packing number of X . In contrast to the chromatic and clique numbers, the packing number can be approximated with good performance quite easily, using a simple kind of “greedy” procedure known as the *first-fit* packing algorithm [4].

3 Basic Theory

It is well-known that the “classical” channel assignment problem in analog broadcast networks can be formulated as a (generalized) graph coloring problem. For this purpose, the network is given by a graph $G = (V, E)$ whose vertices $v \in V$ represent the transmitters. Two vertices v and w are connected by an edge $vw \in E$ if the corresponding transmitters may interfere. In the simplest case, a valid channel assignment then corresponds to a coloring f of G with colors taken from the set of available channels. This graph-theoretic model of channel assignment planning, which was introduced by Hale [8], has proved very useful since it allows us to represent arbitrary network structures arising in real-world problems, and not only the idealized regular grids which are still commonly employed in broadcast network planning by practitioners.

In DAB ensemble planning we are also concerned with the service ensembles to be transmitted, and hence it is convenient to consider not single transmitters, but rather the (geographical) areas for which a certain service supply is planned. This puts us in full control of the subnets which should be considered as single entities transmitting the same ensembles over the same channels; we can always break down the model to the level of single transmitters if this is necessary. We make no additional assumptions about the (geometric) properties of the supply areas; thus areas may be connected or unconnected, may intersect or even contain each other. In practice, supply areas are usually described by collections of polygons on the earth’s surface, and interference relationships are then defined using distance constraints. However, the graph-theoretic approach is also applicable to more realistic models which also take into account morphographic and topographic aspects, such as the models emanating from wave propagation analysis.

Thus in our model an instance of the ensemble planning problem consists of the following items:

- a set S of *services*, where each service $s \in S$ has a positive *bandwidth* μ_s ;
- the *area graph* $G = (V, E)$, where V is the set of areas and the edge set E is interpreted as the interference relationship between areas;